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by

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DISCUSSION PAPER

Pricing of currency options in credible exchange rate target zones: An extension and an alternative valuation approach.

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Abstract

The paper examines pricing of options on target zone exchange rates. The pricing model of Dumas, Jennergren and Näslund (1993) is extended to asymmetric burden sharing in the defence of the target zone. This extension is relevant for various realistic set-ups, such as unilateral target zones. The paper also introduces an alternative pricing model that, in the tradition of Black and Scholes (1973), starts from geometric Brownian motion in which, however, the target zone limits are explicitly taken account of. This approach has a strong appeal from the practical point of view as it is less demanding in terms of required pricing inputs. This, however, goes at the cost of ignoring target zone nonlinearities. Simulations show that the simpler alternative model in most relevant cases moderately underprices by 1% to 3%.

Keywords: (geometric) Brownian motion, exchange rates, option pricing, probability densities, reflecting barriers, target zones.

JEL classification: F31, F33.

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1 Introduction

The international monetary system is characterized by various types of exchange rate target zones. Arrangements can be explicit, such as for instance the Exchange Rate Mechanism (ERM), when monetary authorities announce the magnitude of the fluctuation range. Also silent or implicit target zones are to be found. The Louvre-Accord is perhaps the best-known example of this type in which no official statement on the width of the target zone was given.[1] Also intermediate types exist. For instance, monetary authorities can pursue narrower implicit zones within explicit zones. The existence of credible explicit, implicit or intermediate types of target zones inevitably has to affect pricing of exchange rate derivatives.

The currency option pricing model of Garman and Kohlhagen (1983), that is widely used in the foreign exchange market, can not take account of target zones as it is built upon the assumption of an unrestricted stochastic process. More in particular, the exchange rate is assumed to follow geometric Brownian motion without bounds or the exchange rate can take on values over the entire positive half-line. However, as noted in Dumas, Jennergren and Näslund (DJN) (1993), currency dealers are aware of and do take into account (implicit) target zones. This paper, therefore, examines more closely currency option pricing models that explicitly take the existence of credible target zone arrangements into consideration. The paper extends the pricing model of DJN and introduces an alternative model that is based upon the assumption of reflected geometric Brownian motion.

The first novel aspect of this paper is the extension of the DJN-model to asymmetric burden sharing in the defence of the target zone. The DJN-model assumes that the burden of intervention is shared equally between the domestic and the foreign country. This set-up, however, can easily be extended towards asymmetry. Such an extension is of theoretical and practical relevance as it is the natural modelling choice for several types of target zones. A first example is the unilateral target zone, such as the ECU-pegs of Sweden, Norway and Finland in the period 1990-1992. The burden of intervention is then entirely laid upon the country that chooses to peg.[2] This implies that only the domestic interest rate has to adjust in order to keep the interest rate differential in line with the requirements of the peg. Also multilateral systems such as the ERM can be characterized by asymmetries. The intentions embedded in the agreement of 1979 that created the ERM clearly spoke of equal burden sharing. However, the Bundesbank immediately obtained the (unofficial) right to abstain from intervention if this were to threaten its duty to safeguard the value of the German currency.[3] Whether

[1] See Funabashi (1988) for a detailed account of the Louvre-Accord.

[2] A small note is to be made as the pegs of Norway and Finland were accompanied by (limited) credit facilities vis-à-vis the members of the ERM. In view of their limited amount one could safely assume absence of burden sharing.

[3] See Funabashi (1988), p. 122-123.

or not the ERM behaved in a (completely) asymmetric way, i.e. in line with the German dominance hypothesis, is subject to conflicting views in the empirical literature.[4] However, the months that preceded the widening of fluctuation bands in 1993 were clearly characterized by asymmetric burden sharing. German monetary authorities were confronted by inflationary tensions caused by the huge transfers and increased demand in the wake of German unification and defined monetary policy solely in terms of domestic requirements. Yet, the recession in other European countries called for lower German interest rates. Or, the defence of the target zone fell upon those countries that already were in a recession. Different interest rate levels and responses thus will emerge depending on the nature of burden sharing. Asymmetric burden sharing then will also affect option prices. This can intuitively be explained by noting that the domestic interest rate has a key position in discounting pay-offs of the option.

Second, the paper proceeds by introducing an alternative pricing model. The model is built upon geometric Brownian motion and therefore is akin to the valuation tradition started by Black and Scholes (1973). We, however, explicitly restrict the underlying stochastic process to the fluctuation range offered by the target zone. At the target zone boundaries, the exchange rate will be instantaneously reflected back into the target zone. The model, therefore, can be referred to as the reflected geometric Brownian motion (RGBM) currency option model. From a practical point of view, such a model is easier to implement but this goes at the cost of theoretical weaknesses. It has the strong practical advantage that pricing requires only one unknown input, namely the instantaneous standard deviation of the exchange rate. The DJN-model, on the contrary, requires multiple unknown inputs such as the drift and volatility of the fundamental and the level of the sensitivity parameter.[5] But, the RGBM-model is not constructed within an explicit target zone model. More in particular, the DJN-model, that is explicitly based on the Krugman model, guarantees that the exchange rate does not jump upon intervention. This results from the so-called smooth pasting conditions which renders the model to be arbitrage-free as the no-expected-arbitrage-profits requirement is fulfilled.[6] Imposing the stochastic process of RGBM directly upon the exchange rate as pursued within the RGBM-model will inevitably lead to jumps in the exchange rate (reflection at the boundaries). The RGBM-model also ignores target zone nonlinearities. Veestraeten (2000b) concluded that the presence of nonlinearities could not be rejected for the four ERM-currencies that were examined. This raises the question whether the detected nonlinearities are economically relevant. In other words, simulations will have

[4] See for instance von Hagen and Fratianni (1990), Herz and Röger (1992), Uctum (1999) and Ma and Kanas (2000) and the references given therein.

[5] These concepts will be defined and discussed in more detail in Section 2.

[6] See Krugman (1979,1991) and Flood and Garber (1991) for a detailed discussion of this requirement that originates from the literature on speculative attacks.

to reveal the extent to which neglecting nonlinearities significantly biases option prices. The DJN-model also endogenously determines interest rates in accordance with the target zone literature, whereas the RGBM-model assumes constant interest rates. The model of DJN therefore possesses superior theoretical properties, whereas the RGBM-model requires less pricing inputs. We therefore present simulations in order to gauge pricing accuracy of the RGBM-model. Anticipating results, we detect that the RGBM-model moderately underprices by 1% to 3% with respect to the theoretically superior DJN-model. This bias is fairly limited when compared to the overpricing bias of the model of Garman and Kohlhagen (1983) that amounts to around 200% as shown in DJN.

The paper is organized as follows. Section 2 derives the option pricing model of DJN and extends the model towards asymmetric burden sharing. The RGBM-model is developed in Section 3. Section 4 presents simulations on the impact of asymmetric burden sharing on option prices and discusses pricing accuracy of the RGBM-model. Section 5 concludes.

2 The option pricing model of Dumas, Jennergren and Näslund (1993)

The currency option pricing model of DJN is built upon the canonical target zone model of Krugman (1991). The latter model proceeds in two steps. First, the dynamics of the fundamental variable are specified. In the second step, fundamentals are linked to the exchange rate via the basic asset pricing model. Details on the derivation of the Krugman model can be found in for instance Krugman (1991), Svensson (1991) and Delgado and Dumas (1992). We therefore limit ourselves to a brief overview of the main modelling choices. Subsequently, we derive the DJN-model in which we closely follow the derivations presented in DJN. We then extend the DJN-model by allowing for asymmetric burden sharing in the determination of domestic and foreign interest rates.

The (logarithm of the) fundamental variable, f, is assumed to follow regulated Brownian motion with drift. Its evolution will be constrained by the existence of a lower and an upper band limit, $\underline{f}$ and $\overline{f}$. At these boundaries, the fundamental is instantaneously reflected back into its fluctuation band. The stochastic process for f is therefore termed reflected or regulated Brownian motion and can be written as:

$$df = \mu dt + \sigma dz\left(t\right) + dL - dU, \tag{1}$$

where dL and dU are infinitesimal regulators that are activated when the fundamental reaches either of the boundaries.

The log exchange rate, s, and its driving fundamental are linked through the log-linear asset pricing equation:

$$s = f + \alpha \frac{E[ds]}{dt}, \tag{2}$$

with $\alpha > 0$ and E denoting the expectations operator.[7] Thus, the exchange rate equals the sum of the fundamental and a factor proportional to its own expected change.

Expressing the exchange rate as an explicit function of the fundamental and assuming this function to be twicely differentiable in f allows for the application of Itô's lemma. The expectations term in Eq. (2) then is:

$$\frac{E[ds\,(f)]}{dt} = \mu \frac{ds\,(f)}{df} + \frac{1}{2}\sigma^2 \frac{d^2 s\,(f)}{df^2}. \tag{3}$$

The interest rate differential is obtained through assuming uncovered interest rate parity or

$$(r_d - r_f) = \frac{E[ds]}{dt},$$

which by Eq. (3) equivalently can be written as

$$(r_d - r_f) = \mu \frac{ds\,(f)}{df} + \frac{1}{2}\sigma^2 \frac{d^2 s\,(f)}{df^2}. \tag{4}$$

Before proceeding it is useful to discuss the assumption of uncovered interest rate parity. The interest rate differential is assumed to reflect only the expected exchange rate change. Risk premia are presumed to be absent. This assumption seems acceptable within a model that is built upon the assumption of credibility of the target zone. Moreover, Svensson (1992) argues that even in the presence of realignment risk, risk premia are very small and negligible.

Eq. (4) indicates that the interest rate differential in the DJN-model is determined endogenously. Thus, the interest rate differential will depend on the position of the exchange rate within its fluctuation range. This clearly is a desirable feature as argued in Krugman (1991) and discussed in more detail in Svensson (1991).

So far, we do not have information on the level of the domestic and the foreign interest rates. Knowledge of the domestic interest rate level will become necessary when solving the partial differential equation that arises in option pricing. We follow the modelling choice of DJN who define domestic and foreign interest rates around an exogenous central interest rate, r. In contrast to their assumption of symmetry, we allow for asymmetric burden sharing by introducing the parameter β:[8]

$$r_d \;=\; r + \beta \left(\mu \frac{ds\,(f)}{df} + \frac{1}{2}\sigma^2 \frac{d^2 s\,(f)}{df^2} \right),$$

[7] Time subscripts are omitted for notational brevity. The exchange rate is defined as the number of domestic currency units required for one unit of the foreign currency.

[8] Ekvall, Jennergren and Näslund (1995) present a similar expression but do not discuss it in terms of the potential of asymmetric burden sharing. Moreover, in their simulations they restriction attention to the value $\frac{1}{2}$, i.e. the case of symmetry.

$$r_f = r - (1-\beta)\left(\mu \frac{ds(f)}{df} + \frac{1}{2}\sigma^2 \frac{d^2 s(f)}{df^2}\right), \tag{5}$$

with $0 \leqslant \beta \leqslant 1$. The DJN-model assumes $\beta = \frac{1}{2}$ which induces symmetric burden sharing as will be graphically illustrated below. Complete asymmetry arises for β being equal to 0 or 1. Incompletely and asymmetrically distributed burden sharing results for values different from $\frac{1}{2}$ and the two endpoints of the interval.

We now illustrate the impact and role of the factor β. Panels (a), (b) and (c) in Figure 1 depict domestic and foreign interest rates for β equal to 0, $\frac{1}{2}$ and 1, respectively. The exchange rate band limits are set at 1.1020 and 1.1521.[9] The coefficients μ and σ equal 0 and 0.1, respectively. The sensitivity parameter α is assumed to be 0.7 and the central interest rate r amounts to 0.08. The horizontal axis specifies the exchange rate band. The solid line represents the domestic interest rate, whereas the foreign interest rate is given by the dotted line.

The burden sharing parameter β equals 0 in panel (a). Subtracting the foreign interest rate from the domestic interest rate, i.e. calculating the interest rate differential, illustrates the well-known prediction from the target zone literature that the level of the exchange rate and the interest rate differential are negatively linked together. Indeed, the exchange rate is expected to return to the centre of the band when it is situated close to its upper limit and the interest rate differential therefore has to decrease. For β equal to zero, the domestic interest rate will be constant. The foreign interest rate has to completely carry the burden of defending the target zone. This set-up would arise when the foreign country unilaterally pegs its currency to the domestic currency. This situation would also result when the domestic country is the centre of an asymmetrically functioning multilateral target zone.

Insert Figure 1.

Panel (b) depicts interest rates for $\beta = \frac{1}{2}$. The response in the interest rate differential now will be brought about by symmetrical movements in both the domestic and the foreign interest rates. Moreover, the reaction in the two interest rates aims in the same direction in the sense of widening the interest rate differential.

The burden sharing parameter β is set at unity in panel (c). The foreign interest rate will be independent of the position of the exchange rate within its target zone as the foreign country is not required or willing to share part of the burden of intervention. As exchange rates have two sides, the examples mentioned in the discussion of panel (a) can simply be inverted.

[9] These values correspond to the $\pm 2.25\%$ target zone of the Dutch guilder vis-à-vis the German mark in the latest stage of the ERM before the start of the European Monetary Union.

After digressing on the way interest rates are modelled we now derive the option pricing model. The model is explicitly developed for the fundamental moving between its fluctuation limits. The presence of the target zone, and thus smooth pasting, will be introduced later through the appropriate boundary conditions.
The dynamics of the log exchange rate are obtained through expressing the exchange rate as an explicit function of the fundamental and applying Itô's lemma:

$$ds(f) = \left(\frac{1}{2}\sigma^2 \frac{d^2 s(f)}{df^2} + \mu \frac{ds(f)}{df}\right) dt + \sigma \frac{ds(f)}{df} dz(t).$$

In the next step, we specify the dynamics of the exchange rate level S in order to prepare the ground for the application of the familiar arbitrage relations. Defining $S = \exp(s)$ and applying Itô's lemma gives:

$$dS = \mu_S(S) S dt + \sigma_S(S) S dz(t), \text{ with} \tag{6}$$

$$\mu_S(S) = \frac{1}{2}\sigma^2 \frac{d^2 s(f)}{df^2} + \mu \frac{ds(f)}{df} + \frac{1}{2}\left(\frac{ds(f)}{df}\sigma\right)^2,$$

$$\sigma_S(S) = \frac{ds(f)}{df}\sigma,$$

where $\mu_S(S)$ and $\sigma_S(S)$ denote the instantaneous drift and diffusion coefficients of the stochastic process of S. We proceed by deriving similar expressions for the stochastic process of the currency option. Define $C(S,\tau)$ as the domestic currency price of a call option specified in terms of the domestic market. $C(S,\tau)$ is a function of the exchange rate level, S, and the time until expiration, τ. Application of Itô's lemma yields:

$$dC(S,\tau) = \mu_C(S,\tau) C(S,\tau) dt + \sigma_C(S,\tau) C(S,\tau) dz(t), \text{ with} \tag{7}$$

$$\mu_C(S,\tau) = \frac{\left(\frac{1}{2}\sigma^2 \frac{d^2 s(f)}{df^2} + \mu \frac{ds(f)}{df} + \frac{1}{2}\left(\frac{ds(f)}{df}\sigma\right)^2\right) S \frac{\partial C(S,\tau)}{\partial S} - \frac{\partial C(S,\tau)}{\partial \tau}}{C(S,\tau)}$$

$$+ \frac{\frac{1}{2}\left(\frac{ds(f)}{df}\sigma\right)^2 S^2 \frac{\partial^2 C(S,\tau)}{\partial S^2}}{C(S,\tau)}$$

$$\sigma_C(S,\tau) = \frac{\frac{ds(f)}{df}\sigma S \frac{\partial C(S,\tau)}{\partial S}}{C(S,\tau)},$$

with $\mu_C(S,\tau)$ and $\sigma_C(S,\tau)$ being the instantaneous coefficients of the option price process.

The usual arbitrage relations require the domestic investor to be indifferent between investing in the exchange rate and in the option, or[10]

$$\frac{\mu_S(S) - \left(r_d - r_f\right)}{\sigma_S(S)} = \frac{\mu_C(S,\tau) - r_d}{\sigma_C(S,\tau)} = 0. \tag{8}$$

The first and second terms in Eq. (8) equal zero in view of the assumption of risk-neutrality. In words, the expected drift of the exchange rate coincides with its risk-neutral equivalent which is the interest rate differential. Similarly, the drift of the option has to be equal to the domestic risk-free interest rate.

Plugging Eqs. (6) and (7) into Eq. (8), expressing the interest rate differential in terms of Eq. (4) and the domestic interest rate via Eq. (5) then gives the following partial differential equation:

$$\begin{aligned}
&\frac{1}{2}\left(\frac{ds(f)}{df}\sigma\right)^2 S^2 \frac{\partial^2 C(S,\tau)}{\partial S^2} + \left(\frac{1}{2}\sigma^2 \frac{d^2 s(f)}{df^2} + \mu \frac{ds(f)}{df}\right) S \frac{\partial C(S,\tau)}{\partial S} - \\
&\frac{\partial C(S,\tau)}{\partial \tau} - \left(r + \beta\left(\mu \frac{ds(f)}{df} + \frac{1}{2}\sigma^2 \frac{d^2 s(f)}{df^2}\right)\right) C(S,\tau) = 0.
\end{aligned} \tag{9}$$

Eq. (9) shows that the option price will depend on the degree of burden sharing, β. Different prices thus result for different types of target zone arrangements as the domestic interest rate will respond differently across regimes.

The DJN-model is consistent with the (unrestricted) model of Garman and Kohlhagen (1983) when the target zone grows infinitely wide, i.e. for $\underline{f} \rightarrow -\infty$ and $\overline{f} \rightarrow +\infty$. Eq. (9) then reduces to the valuation equation of Garman and Kohlhagen (1983):

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C(S,\tau)}{\partial S^2} + (r_d - r_f) S \frac{\partial C(S,\tau)}{\partial S} - \frac{\partial C(S,\tau)}{\partial \tau} - r_d C(S,\tau) = 0. \tag{10}$$

Note that σ in Eq. (10) denotes the instantaneous standard deviation of the exchange rate which is assumed to follow the following stochastic process $dS = \mu S dt + \sigma S dz(t)$. The coefficients of the second and fourth terms in Eq. (9) can be replaced by $(r_d - r_f)$ and r_d. Subsequently, we have to show that the limit of $\left(\frac{ds(f)}{df}\sigma\right)$, the coefficient of the first term, equals σ, where the diffusion coefficients relate to the log fundamental and the log exchange rate, respectively. Under the free float, the exchange rate solution can be written as $s = f + \alpha\mu$ and the derivative of s to f is unity. As α and μ are assumed to be constant, the diffusion coefficients of s, in the specification of Garman and Kohlhagen (1983), and f will be identical.

Before proceeding we discuss the relation between the pricing models of DJN, Garman and Kohlhagen (1983) and the RGBM-model that will be developed in Section 3. The DJN-model accounts for the curvature effect and the boundaries implied by the presence of the

[10] See for instance Garman and Kohlhagen (1983).

target zone arrangement. The model of Garman and Kohlhagen (1983) takes neither of these two target zone characteristics into consideration. This can be seen from the differing nature of the coefficients of the first terms in Eqs. (10) and (9), from the definition of the interest rate (differential) and the presence of boundary conditions in the case of the DJN-model. The RGBM-model differs from the model of Garman and Kohlhagen (1983) by explicitly taking into account the target zone boundaries.[11] In other words, the model of Garman and Kohlhagen (1983), when applied to target zone exchange rates, reveals two theoretical weaknesses, notably the absence of the curvature effect and the boundary conditions. The RGBM-model introduces boundary conditions but will not be able to embed the resulting curvature effects. The DJN-model is theoretically superior to the prior two models as both curvature and boundaries are built in.

In what follows, the valuation equation in Eq. (9) will be solved numerically. For reasons of convenience it will be expressed in terms of the fundamental f. This requires a double transform. First, the option contract is defined in terms of the log exchange rate s. Second, the pricing equation is rewritten through assuming the option contract to be a direct function of the fundamental f and the time to maturity τ, i.e. $C(f,\tau)$. This yields the following partial differential equation:

$$\begin{aligned}&\frac{1}{2}\sigma^2\frac{\partial^2 C(f,\tau)}{\partial f^2}+\left(\mu-\frac{1}{2}\sigma^2\frac{ds(f)}{df}\right)\frac{\partial C(f,\tau)}{\partial f}-\frac{\partial C(f,\tau)}{\partial \tau}-\\&\left(r+\beta\left(\mu\frac{ds(f)}{df}+\frac{1}{2}\sigma^2\frac{d^2 s(f)}{df^2}\right)\right)C(f,\tau)=0.\end{aligned}\tag{11}$$

Eq. (11) is subject to one initial condition and two boundary conditions. The intial condition is the pay-off function of the European call option, namely

$$C(f,0)=\max\left[0,\exp(s(f))-K\right],\tag{12}$$

where K denotes the exercise price. The boundary conditions follow from the smooth pasting conditions. Tangency must hold for the log exchange rate with respect to the boundaries of the fluctuation range for the fundamental. This requirement logically extends to S and consequently also the option price $C(S,\tau)$ has to be tangent to the boundaries of the fundamental band. The double transform that led from Eq. (9) to Eq. (11) allows us to express the boundary conditions in terms of $C(f,\tau)$:

$$\left.\frac{\partial C(f,\tau)}{\partial f}\right|_{f=\overline{f}}=0 \text{ and } \left.\frac{\partial C(f,\tau)}{\partial f}\right|_{f=\underline{f}}=0.\tag{13}$$

[11] The pricing equations for both models are identical. But in the case of the RGBM-model the partial differential equation is to be solved subject to two boundary conditions. Section 3, however, will follow a different strategy by specifying the conditional density function for the exchange rate and utilizing the risk-neutral valuation approach.

Eqs. (11), (12) and (13) specify the option pricing model of DJN extended to the possibility of unequal burden sharing.

3 The reflected geometric Brownian motion currency option model

The pricing model of DJN has two desirable features. First, the model is embedded in a simple but explicit target zone model that satisfies the no-expected-arbitrage-profits condition. Second, interest rates are derived endogenously which is also in line with (intuitive) prerequisites from the target zone literature. The model, however, has a serious drawback from the point of view of practical application. The number of pricing inputs is quite extensive and requires priors on the level of the sensitivity parameter α as well as the coefficients of the stochastic process of the fundamental. Inference on these parameters is far from easy as the extensive literature on the level of α already indicates.

One could therefore ask whether a pricing model built around reflected geometric Brownian motion could be a workable alternative. Such a model has the intuitive advantage of being a straightforward extension of the well-known pricing framework of Black and Scholes (1973). Most important for practical purposes is the fact that, similarly to Black and Scholes (1973), only one unknown parameter will be required in pricing, namely the instantaneous standard deviation of the underlying asset. Note that the RGBM-model arises as the limit case of DJN when evaluating the limit for α going to zero.[12] This limit, however, brings us to the theoretical drawbacks of the RGBM-model. Nonlinearities in the exchange rate are ruled out as markets are no longer forward-looking. Also, smooth pasting will be absent or the no-expected-arbitrage-profits condition can be violated. Moreover, the introduction of the conditional density into the pricing framework, as pursued below, will assume constant interest rates.

It remains, however, to be answered to what extent these theoretical weaknesses of the RGBM-model are of practical importance. First of all, evidence of nonlinearities is weak or absent in the empirical target zone literature. Furthermore, and related to the first remark, exchange rates tend to spend much less time near the edges of the target zone as predicted by theoretical models. Moreover, option pricing literature does not point to a significant (practical) advantage of stochastic over constant interest rates. Or, approximating the DJN-model by the RGBM-model may be an interesting alternative.

[12] The stochastic process for s then simply coincides with the stochastic specification of f as can be seen from Eq. (2). Or, s follows reflected Brownian motion and S follows reflected geometric Brownian motion.

This section develops the RGBM-model. The first subsection specifies the conditional density function of reflected geometric Brownian motion. The second subsection proceeds by calculating European call option prices via the risk-neutral valuation approach.

3.1 The conditional density of reflected geometric Brownian motion

The exchange rate level S is assumed to follow geometric Brownian motion. Superimposed on this process are two boundaries at which the exchange rate will be instantaneously reflected. Denote the upper and lower boundary by $\overline{S}$ and $\underline{S}$, respectively. We have:

$$dS = \mu S dt + \sigma S dz\left(t\right),$$

between the boundaries and instantaneous reflection at $\overline{S}$ and $\underline{S}$. The transform $s = \ln S$ yields the following stochastic process for the log exchange rate:

$$ds = (\mu - \frac{1}{2}\sigma^2)dt + \sigma dz\left(t\right),$$

with reflection at the log of the original boundaries, i.e. at $\overline{s}$ and $\underline{s}$. The solution to this stochastic differential equation is the transition or conditional probability density function of s that is denoted by $p\left(s, t; s_0, t_0\right)$. It specifies the density with respect to the intial (or present) state s_0 and the intial (or present) point of time t_0. The density function for arithmetic Brownian motion with drift between two reflecting boundaries is derived in Veestraeten (2000a). Appropriately adapting the drift and the boundaries in his Eq. (11) gives:

$$\begin{aligned}
p\left(s,t;s_0,t_0\right) = &\sum_{n=-\infty}^{+\infty}\left\{\frac{1}{\sigma\sqrt{2\pi\left(t-t_0\right)}}\exp\left(\frac{2\gamma n\left(\underline{s}-\overline{s}\right)}{\sigma^2}\right)\exp\left(-\frac{\left(s+2n(\overline{s}-\underline{s})-s_0-\gamma\left(t-t_0\right)\right)^2}{2\sigma^2\left(t-t_0\right)}\right)\right\}\\
&+\sum_{n=-\infty}^{+\infty}\left\{\frac{1}{\sigma\sqrt{2\pi\left(t-t_0\right)}}\exp\left(-\frac{2\gamma\left(n\overline{s}-\left(n+1\right)\underline{s}+s_0\right)}{\sigma^2}\right)\right.\\
&\qquad\left.\exp\left(-\frac{\left(2n\overline{s}-2\left(n+1\right)\underline{s}+s_0+s-\gamma\left(t-t_0\right)\right)^2}{2\sigma^2\left(t-t_0\right)}\right)\right\}\\
&-\frac{2\gamma}{\sigma^2}\sum_{n=0}^{+\infty}\left\{\exp\left(\frac{2\gamma\left(n\overline{s}-\left(n+1\right)\underline{s}+s\right)}{\sigma^2}\right)\left[1-\Phi\left(\frac{\gamma\left(t-t_0\right)+2n\overline{s}-2\left(n+1\right)\underline{s}+s_0+s}{\sigma\sqrt{t-t_0}}\right)\right]\right\}\\
&+\frac{2\gamma}{\sigma^2}\sum_{n=0}^{+\infty}\left\{\exp\left(\frac{2\gamma\left(n\underline{s}-\left(n+1\right)\overline{s}+s\right)}{\sigma^2}\right)\Phi\left(\frac{\gamma\left(t-t_0\right)-2\left(n+1\right)\overline{s}+2n\underline{s}+s_0+s}{\sigma\sqrt{t-t_0}}\right)\right\},
\end{aligned}$$

where $\Phi\left(z\right) = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{z}\exp\left(-\frac{1}{2}y^2\right)dy$ denotes the cumulative standard normal distribution function and $\gamma = \mu - \frac{1}{2}\sigma^2$. Next, we transform this conditional density function into the density for the original variable S. Replacing t and t_0 by T and t, respectively and applying a simple change of variable then yields:[13]

$$
\begin{aligned}
p\left(S_T,T;S_t,t\right) = \sum_{n=-\infty}^{+\infty} & \left\{ \frac{1}{S_T\sigma\sqrt{2\pi\left(T-t\right)}} \exp\left(\frac{2\gamma n\left(\underline{s}-\overline{s}\right)}{\sigma^2}\right)\right. \\
& \left. \exp\left(-\frac{\left(\ln S_T + 2n(\overline{s}-\underline{s}) - \ln S_t - \gamma\left(T-t\right)\right)^2}{2\sigma^2\left(T-t\right)}\right)\right\} \\
+ \sum_{n=-\infty}^{+\infty} & \left\{ \frac{1}{S_T\sigma\sqrt{2\pi\left(T-t\right)}} \exp\left(-\frac{2\gamma\left(n\overline{s}-\left(n+1\right)\underline{s}+\ln S_t\right)}{\sigma^2}\right)\right. \\
& \left. \exp\left(-\frac{\left(2n\overline{s}-2\left(n+1\right)\underline{s}+\ln S_t+\ln S_T-\gamma\left(T-t\right)\right)^2}{2\sigma^2\left(T-t\right)}\right)\right\} \\
-\frac{2\gamma}{\sigma^2}\sum_{n=0}^{+\infty} & \left\{\frac{1}{S_T}\exp\left(\frac{2\gamma\left(n\overline{s}-\left(n+1\right)\underline{s}+\ln S_T\right)}{\sigma^2}\right)\right. \\
& \left. \left[1-\Phi\left(\frac{\gamma\left(T-t\right)+2n\overline{s}-2\left(n+1\right)\underline{s}+\ln S_t+\ln S_T}{\sigma\sqrt{T-t}}\right)\right]\right\} \\
+\frac{2\gamma}{\sigma^2}\sum_{n=0}^{+\infty} & \left\{\frac{1}{S_T}\exp\left(\frac{2\gamma\left(n\underline{s}-\left(n+1\right)\overline{s}+\ln S_T\right)}{\sigma^2}\right)\right. \\
& \left. \Phi\left(\frac{\gamma\left(T-t\right)-2\left(n+1\right)\overline{s}+2n\underline{s}+\ln S_t+\ln S_T}{\sigma\sqrt{T-t}}\right)\right\}.
\end{aligned}
\tag{14}
$$

Eq. (14) specifies the conditional density function for regulated geometric Brownian motion with drift between the two reflecting barriers $\underline{S}$ and $\overline{S}$.

Examining the unrestricted domain gives:

$$
\lim_{\substack{\underline{S}\to 0\\ \overline{S}\to+\infty}} \left[p\left(S_T,T;S_t,t\right)\right] = \frac{1}{S_T\sigma\sqrt{2\pi\left(T-t\right)}}\exp\left(-\frac{\left(\ln S_T-\ln S_t-\left(\mu-\frac{1}{2}\sigma^2\right)\left(T-t\right)\right)^2}{2\sigma^2\left(T-t\right)}\right),
\tag{15}
$$

which, as required, is the unrestricted distribution function given in for instance Aitchison and Brown (1973).[14]

[13] The necessary conditions can be found in Theorem 11 in Mood, Graybill and Boes (1974), p. 200.

[14] In deriving this result, we considered in turn each of the four terms in Eq. (14). The limit for the first term equals the density in Eq. (15) as all the elements in the sum for which n differs from 0 will be zero as their limit can be expressed as $\exp(-\infty)$. All elements in the second term vanish for the same reason. The third term also completely vanishes as each of the elements in the sum, in a stylized manner, can be rewritten

3.2 Risk-neutral valuation of European call options

We now will work towards the valuation formula for the European call option. In line with the approach of Garman and Kohlhagen (1983) we assume interest rates to be constant. Risk-neutral valuation involves switching from the original probability measure to the risk-neutral probability measure. This boils down to replacing the original drift factor μ by its risk-neutral equivalent which is the interest rate differential.[15] Thus, μ in the definition of the factor γ in Eq. (14) is to be replaced by $(r_d - r_f)$. The state-dependent pay-off of the European call option with exercise price K can be written as $\max[0, S_T - K]$ where $\underline{S} \leqslant K \leqslant \overline{S}$. The price of the European call option contract at time t then is:

$$C(S,\tau) = \exp(-r_d(T-t)) \int_{\underline{S}}^{\overline{S}} \max[0, S_T - K]\, p(S_T, T; S_t, t)|_{\mu = r_d - r_f}\, dS_T. \qquad (16)$$

Three notes are to be made. First, valuation is pursued under the assumption of risk neutrality or absence of risk premia. In the case of a narrow target zone such a starting point is warranted in view of the afore-mentioned results of Svensson (1992). Second, we applied the switch between the original probability measure and the risk-neutral measure to a restricted process. The presence of reflecting boundaries does not impair moving between these two measures. Indeed, under the original measure the probability of finding exchange rates outside of the (credible) target zone is zero. This logically requires that the corresponding probability under the equivalent measure also must be zero. Or, the presence and location of reflecting boundaries has no effect on switching between measures. Third, taking the limits to the unrestricted case yields the pricing equation of Garman and Kohlhagen (1983) as the limit of the risk-neutral density equivalent of Eq. (14) yields the risk-neutral density of unrestricted geometric Brownian motion that can be obtained via Eq. (15).

4 Simulations

We first illustrate the impact of different values for the burden sharing parameter on the DJN-option value. These simulations allow us to assess the effect on option prices of differing types of target zone arrangements. The second subsection examines pricing differences between the

as $\lim_{z\to+\infty} [\exp(az)\{1-\Phi(z)\}]$ with a being a positive or negative constant. This term is zero by virtue of l'Hôpital's rule. Also the fourth term entirely vanishes as $\lim_{z\to+\infty} [\exp(az)\,\Phi(z)] = 0$.

[15] Equivalently, the risk-neutral probability distribution function is the solution to the stochastic differential equation $dS = (r_d - r_f)\, Sdt + \sigma Sdz(t)$.

DJN-model and the RGBM-model. The purpose is to evaluate whether the reduction in valuation inputs does not impair pricing accuracy.

4.1 The impact of differing degrees of burden sharing

The effect of differing values for β on prices is assessed for the parameter values that underlie Figure 1. Panels (a), (b) and (c) in Figure 2 depict option prices given $\beta = 0$, $\frac{1}{2}$ and 1, respectively.[16] Prices are multiplied by 1000. Panels (a) and (b) in Figure 3 more clearly detail price differences by subtracting prices under $\beta = \frac{1}{2}$ and $\beta = 1$ from prices of contracts for which β is assumed to be zero. The horizontal axis gives the exchange rate target zone, the lines thus represent the option price functions or the price differences for every possible initial value of the exchange rate.

Insert Figures 2-3.

Panels (a), (b) and (c) illustrate that the boundary conditions in Eq. (13) also cause the option price function to have the familiar S-shape. The economic rationale is simple. The no-expected-arbitrage-profits condition requires the exchange rate not to have discontinuities in its sample path upon intervention in the fundamental. As the option price is functional upon S also the option price is not to jump upon intervention. Otherwise, free lunches would be possible. Tangency also holds for the option price and the S-shape carries over to the option price function as well. The panels show that prices are extremely small. This is due to the combined assumption of a relatively narrow target zone and zero drift in the fundamental.

Figure 3 illustrates the effect of the burden sharing parameter in more detail. Panel (a) depicts the difference in price between option contracts for which $\beta = 0$ and $\beta = \frac{1}{2}$. A value above zero indicates that the contract for which $\beta = 0$ is worth more than the contract characterized by $\beta = \frac{1}{2}$. Panel (b) compares prices for $\beta = 0$ versus $\beta = 1$. Price differences in both panels are only positive for options that are deep out-of-the-money[17], whereas the reverse holds for at- and in-the-money options. Closer examination of the levels in panels (a) and (b) shows that there is a clear order in pricing differences. For out-of-the money options the highest price is to be found for the contract in which $\beta = 0$, followed by the specification in which $\beta = \frac{1}{2}$ and finally $\beta = 1$. The reverse holds for at- and in-the-money options. The explanation is simple and relates to discounting of the future pay-offs. As noted earlier, the level of the burden sharing parameter does not enter in the dynamics of the exchange rate nor affects the interest rate differential. It only appears in the fourth term in

[16] Option prices here and in the remainder of the paper are always denoted in the domestic currency and pertain to contracts specified for the domestic market.

[17] Remember that the exercise price is located in the middle of the exchange rate band.

the pricing equation of Eq. (9) where it steers discounting of future pay-offs. The discount factor is the domestic interest rate that will vary in function of the position of the exchange rate within its target zone. From Figure 1, we know that the domestic interest rate, in case of $\beta = 0$, will be constant at the central interest rate. Equal burden sharing, i.e. $\beta = \frac{1}{2}$, implies that the domestic interest rate exceeds the central interest rate in the lower part of the band and falls below that level in the upper part of the target zone. Finally, for $\beta = 1$ the same is true albeit that differences with respect to the central interest rate are larger (in absolute value). Note, that there will only be a positive pay-off at maturity date when the exchange rate rises above the central parity at which level we have set the exercise price. For options that are deep out-of-the money, this requires the exchange rate to move through a large segment of the lower part of the exchange rate band in order to attain the profit zone. Thus, the domestic interest rate will be strongly above the central interest rate throughout this segment for $\beta = 1$. Or, discounting will embody mostly interest rates that are located far above the central interest rate. In case of β being equal to $\frac{1}{2}$ lower domestic interest rates enter the discount factor. As a result, the option price will exceed the one resulting for $\beta = 1$. Applying the same reasoning for $\beta = 0$ then shows that option prices must decrease the higher the burden sharing parameter β becomes. The reverse order holds for at- and in-the money options. This is explained by the fact that the upper side of the target zones becomes increasingly more important.[18] Domestic interest rates in that segment will be lower for larger values of β. As a result, option prices will be positively linked to the value of β.

Figures 2 and 3 thus reveal that the degree of burden sharing has a (limited) effect upon option prices. Moreover, the impact depends on the position of the exchange rate within its target zone.

4.2 Valuation via the reflected geometric Brownian motion option model[19]

Figures 4-6 present plots of percentage price differences between the DJN- and the RGBM-models. Percentage price differences are defined as:

$$\text{\%-difference} = \frac{C_{RGBM} - C_{DJN}}{C_{DJN}} * 100,$$

where C_{RGBM} and C_{DJN} denote European call option prices calculated on the basis of the RGBM- and the DJN-models, respectively. Positive values indicate that the RGBM-model

[18] One could alternatively argue that the (bell-shaped) conditional probability function will shift to the right when the initial exchange rate grows larger. See Veestraeten (2000b) for more detail on the conditional exchange rate distribution that arises within the Krugman model.

[19] The valuation equation in Eq. (16) can be solved in an analytical manner. The pricing formula and the main steps in its derivation can be obtained from the author. However, the formula is numerically ill-behaving for certain parameter ranges due to the potential of explosively growing exponentials. We therefore solve Eq. (16) via numerical integration.

overprices the contract, negative values point to undervaluations. The three figures share the following parameters. The upper and lower exchange rate bands, as before, are set at 1.1019 and 1.1521, respectively. The exercise price is set at the midpoint of the target zone and the remaining time to maturity is six months.[20] The instantaneous drift of the fundamental equals zero. The constant interest rate differential that is required within the RGBM-model is set at zero. The central interest rate equals 8%. The domestic interest rate in the RGBM-model is also set at 8%. This warrants some explanation as we examine prices for different levels of the burden sharing parameter. When $\beta = 0$, the domestic interest rate in the DJN-model equals the central interest rate and is constant. Setting the domestic interest rate in the RGBM-model at the level of the central interest rate is then a logical choice. If β equals unity the foreign interest rate is constant at the central interest rate. Assuming the interest rate differential to be zero in the RGBM-model then allows us to equate the central and the domestic interest rates. If $\beta = \frac{1}{2}$ we apply the same reasoning, albeit knowing that this holds exactly only when the exchange rate is located in the middle of the band as can be seen from panel (b) in Figure 1. As we assume absence of drift, we feel that extending this equality to the entire band is a reasonable choice.

Insert Figures 4-6.

Figures 4-6 differ with respect to the position of the initial exchange rate which is set at the middle of the lower half, the overall middle and at the center of the upper part of the target zone, respectively. The right horizontal axis depicts values for the sensitivity parameter α that ranges between 0.01 and 5. This range of values is in line with estimation results reported in Veestraeten (2000b). The left horizontal axis gives the grid for the diffusion coefficient σ for the process of the fundamental (DJN-model) and of the exchange rate (RGBM-model).[21] We chose for a range of 0.05 to 0.2 p.a.

Figure 4 shows, as will also be true for Figures 5 and 6, that the level of the burden sharing parameter β has only a limited influence on the nature and the level of pricing differences. Also the sensitivity parameter α does not substantially affect pricing differences. This is not surprising as α governs the extent of nonlinearities. Nonlinearities typically are concentrated near the boundaries of the exchange rate band. As the initial exchange rate is located in the middle of the lower band part and the fundamental exhibits no drift, these regions enjoy low probability mass. Hence, price differences will be fairly insensitive to α, although, and this is hardly visible from the graphs, they grow less positive or more negative. The crucial factor

[20] We performed also simulations for contracts with one year until expiration. Results, however, were qualitatively indistinguishable from those that will be reported below.

[21] As noted earlier, the RGBM-model can be derived from the DJN-model by setting α to zero in which case the stochastic process of the fundamental carries over to the exchange rate. This allows us to use the same value for the diffusion coefficients in the two models.

turns out to be the volatility parameter σ. Pricing differences are initially marginally positive and from a value for σ of around 8% onwards price differences are more or less stable between -2% and -2.5%. For low degrees of variability in the exchange rate the RGBM-model slightly overestimates, whereas for moderate to larger levels for the second moment only moderate degrees of underestimation arise. The mean difference in the three cases is about -2.2%.

When the initial exchange rate is located in the centre of the target zone, as in Figure 5, the RGBM-model always underprices with a mean around -2.5%. The shape of the price difference surface is virtually flat when excluding the lowest levels for α and σ. Thus, for most practical cases the RGBM-model has a small and stable underpricing bias.

The picture is somewhat different if the initial exchange rate is located in the upper part of the exchange rate band as in Figure 6. For low volatility levels, the RGBM-model severely underestimates. However, increasing σ again yields pricing errors around -2% to -3% yielding an overall mean for the pricing difference between -2.8% and -3.1%. The surface again is flat when excluding the lower end of the grid for the dispersion coefficient.

In conclusion, the RGBM-model will underestimate the theoretically correct option price that is given by the DJN-model. Underpricing is typically of the order of 2% to 2.5%. Mispricing is only substantial for low values of the volatility parameter. It is true that target zones typically reduce the volatility parameter, but one can argue that volatility levels as low as 5% p.a. are of only limited practical relevance for option pricing. Options on exchange rates that enjoy such low dispersion are unlikely to be in great demand as the cost of the option may well equal or exceed the (expected) potential loss. For instance, options on the guilder-mark exchange rate during the ERM were, to the best of our knowledge, not extant. On the other hand, options on the lira-mark exchange rate, for which volatility was larger, have always been offered by financial institutions. Therefore, the RGBM-model may well be a worthwhile and practically relevant model for the valuation of currency options on target zone exchange rates. Moreover, as the surface of price differences is virtually flat for relevant parameter values, one could use in practical applications a rule of thumb of adding 2% to 2.5% to RGBM-prices.

Finally, the small price differences also allow for the conclusion that nonlinearities in target zone exchange rates are of limited (economic) relevance for option pricing. On the other hand, taking account of the target zone limits is of the utmost importance for practical applications. Indeed, prices from the model of Garman and Kohlhagen (1983) are typically 200% above theoretically correct prices as shown in DJN.[22]

[22] The figures in DJN assumed σ to equal 0.1. We therefore performed some simulations on the relation between the RGBM-model and the model of Garman and Kohlhagen (1983) for various volatility levels. As earlier, we let σ vary between 0.05 and 0.2. Results showed that Garman and Kohlhagen (1983) overestimates RGBM-prices by 100% to about 1000%. Overpricing monotonically increases in the volatility level. This is due

5 Conclusion

This paper examines currency option pricing within credible target zones. The DJN-model, which is developed within the framework of the target zone model of Krugman (1991), is extended to asymmetric burden sharing in the defence of the target zone. The DJN-model assumed that the burden of intervention is shared equally between countries. Such a set-up, however, is not appropriate in the case of unilateral target zones. The country that pursues the peg cum band arrangement has to bear the entire burden of intervention and its domestic interest rate will be more variable than under a symmetric system. As a result, option prices will differ in function of the degree of burden sharing as future pay-offs are to be discounted at the domestic interest rate. More in particular, for deep out-of-the-money options prices will be higher for lower values of the burden sharing parameter, β. Or, higher prices result when the burden of intervention is smaller for the domestic country. The reverse holds for in- and at-the-money options. Asymmetric burden sharing is, however, not limited to unilateral target zones but can also emerge within multilateral settings such as the ERM. The months that preceded the turmoil of 1993 are an instance where such asymmetric burden sharing took place.

The paper also introduces an alternative framework for the valuation of options on target zone exchange rates. The alternative model is embedded within the risk-neutral valuation approach and requires knowledge of the conditional density function for RGBM. It is akin to the most wide-spread option pricing methodology, namely that of Black and Scholes (1973), as it shares the assumption that the asset price follows geometric Brownian motion. However, the RGBM-model explicitly takes account of the presence of target zone limits. The main advantage of the RGBM-model is that pricing, as in Black and Scholes (1973), requires only inference on one unknown input, namely the instantaneous standard deviation. This feature is of practical relevance as the DJN-model necessitates inference on various additional unknown parameters such as the sensitivity factor α and the drift of the fundamental. The ease in implementation of the RGBM-model goes at the cost of displaying three theoretical drawbacks. The first is related to the fact that jumps in the exchange rate can not be precluded which violates the no-arbitrage condition. Second, target zone nonlinearities are absent. Third, interest rates are assumed to be constant and as such do not depend on the position of the exchange rate within its fluctuation band. We, therefore, present simulations in order to assess the pricing accuracy of the RGBM-model. They reveal that the RGBM-model,

to the fact that higher values for σ, within the unrestricted model, imply larger probability mass for exchange rates above the upper target zone limit. As a result, the pay-off of the option increases and prices will be far above those obtained by the RGBM-model that accounts for the presence of target zone limits. For brevity, we do not include these figures although they can be obtained from the author upon simple request.

in general, possesses an underpricing bias of 1% to 3% when compared to the DJN-model. The extent of this bias is only marginally depending on the level of dispersion, the sensitivity parameter and the burden sharing parameter.[23] Only for rather low dispersion of around or lower than 5% p.a., mispricing can grow to about 7%. As the usefulness and supply of options on such exchange rates may be doubted, we feel that the RGBM-model can serve as an easy-to-use approximation to more elaborate models in most relevant applications.

The relation of RGBM- to DJN-prices also sheds some light on the economic relevance of target zone nonlinearities. The effect on pricing is very small as revealed by the simulations. Thus, target zone nonlinearities, as reported for instance in Veestraeten (2000b), are of limited importance for option pricing. On the other hand, the inclusion of target zone limits in option pricing is crucial as the unrestricted model of Garman and Kohlhagen (1983) returns prices that surpass theoretically correct prices by 200% as indicated by DJN.

The paper can be extended in several directions. A first and straightforward extension is the development of pricing frameworks geared towards one-sided target zones. Second, the concept of asymmetric burden sharing in this paper is very simple in nature but can be given more detail by relating it, for instance, to the position of the exchange rate in the target zone. Both models can also be extended towards mean-reversion. In that way one could take account of intramarginal interventions. This route was taken up in Ekvall, Jennergren and Näslund (1995) for the symmetric burden sharing model of DJN. The RGBM-model can be extended to intramarginal interventions by imposing a reflected Ornstein-Uhlenbeck process. However, the conditional probability density in the case of two-sided reflection has not yet been derived for this process.[24] The above pricing frameworks rely on perfect credibility and thus preclude realignments. The DJN-model was extended to include realignment expectations in Dumas, Jennergren and Näslund (1995). However, that model can also be extended towards asymmetric burden sharing which becomes even more acute when credibility of the peg is not perfect. Realignment risk can be introduced into the RGBM-framework, for instance, through the use of a composite fundamental that includes stochastic devaluation risk as advanced in Bertola and Svensson (1993). The RGBM-model can also be seen as a first step towards pricing of options on exchange rates that are subject to a (credible) crawling peg cum band. The required moving and time-dependent bands can be introduced via linear retaining or straight line barriers, see Park and Schuurmann (1980), Gerber, Goovaerts and De Pril (1981), Park and Beekman (1983) and Teunen and Goovaerts (1994).

[23] Note that the latter two parameters do not enter in the RGBM-model.

[24] Ricciardi and Sacerdote (1987) derive an integral representation for the one-sided reflected Ornstein-Uhlenbeck process. Ball and Roma (1994) use this representation in approximating the conditional density for the two-sided reflected process.

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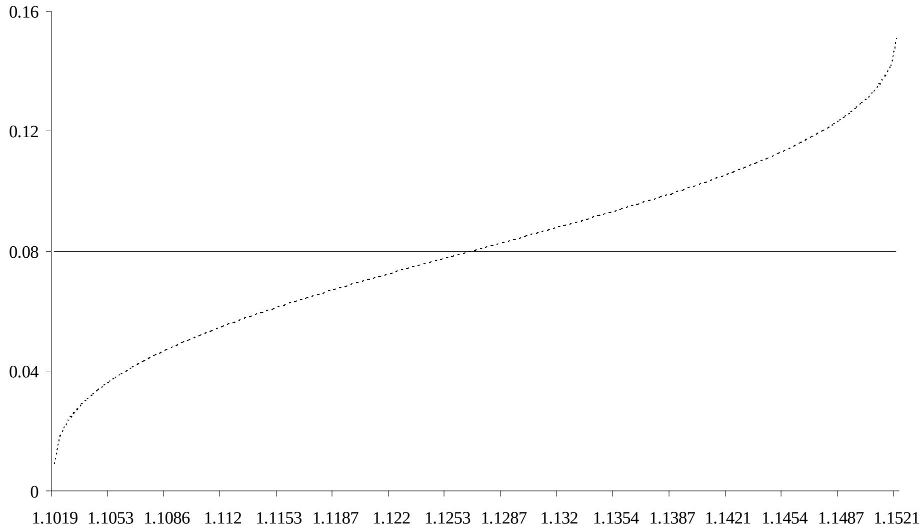
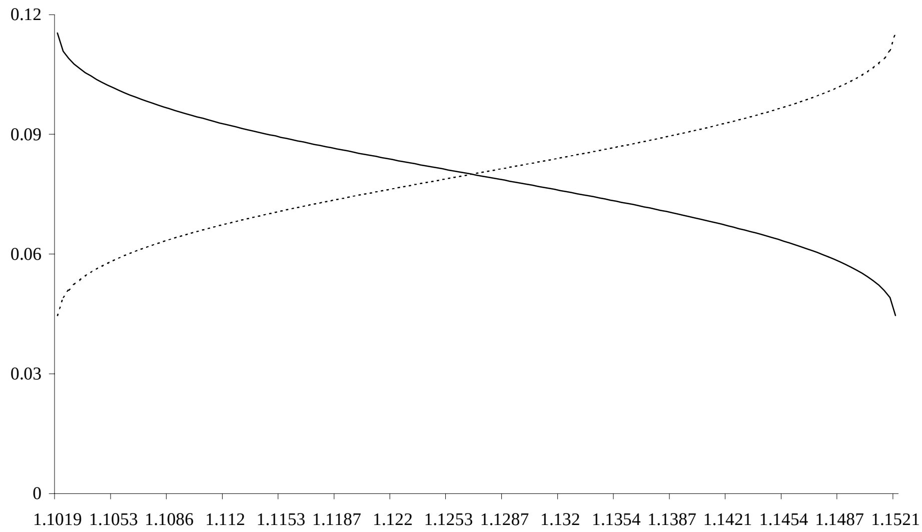
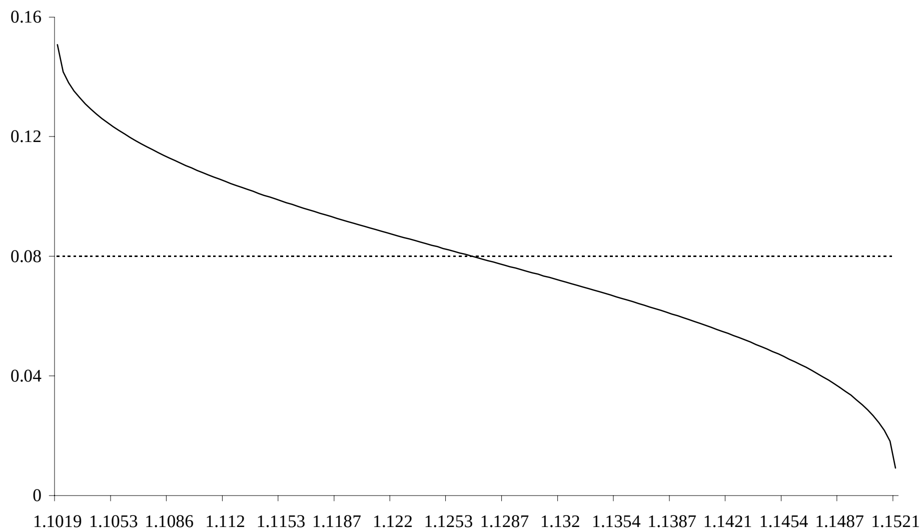
Figure 1: The domestic interest rate (solid line) and the foreign interest rate (dotted line) for $\beta = 0$ (panel a), $\beta = 0.5$ (panel b) and $\beta = 1$ (panel c).



Panel (a)

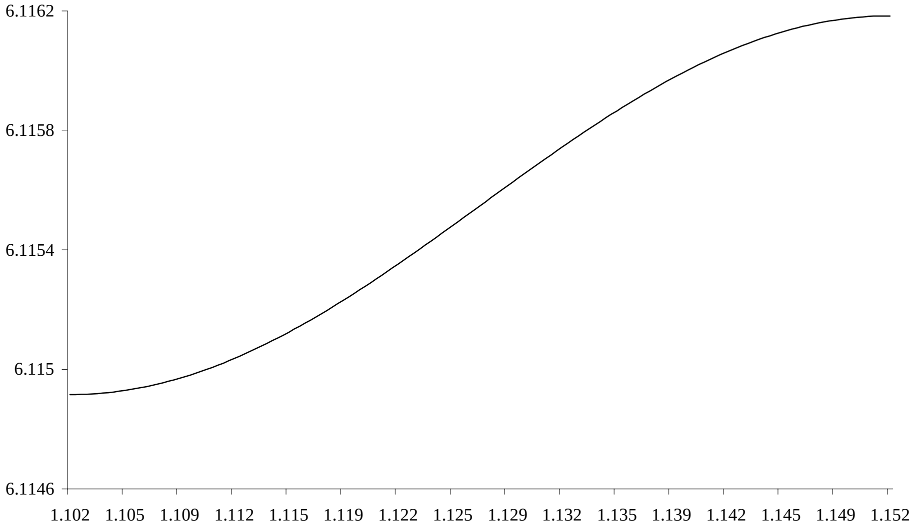


Panel (b)

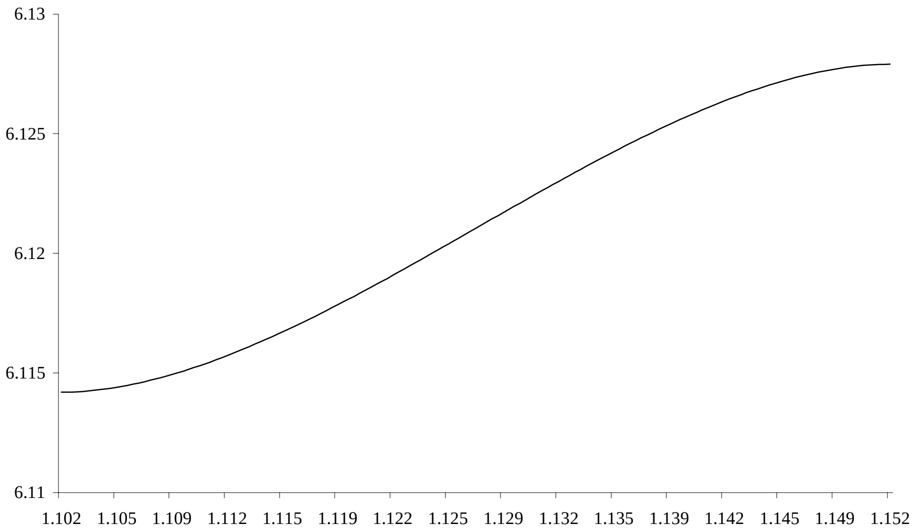


Panel (c)

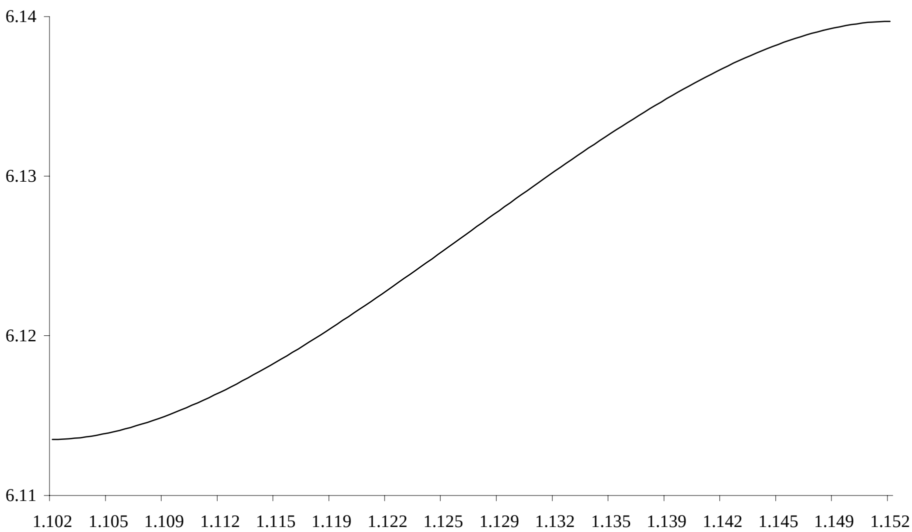
Figure 2: The currency option price function for $\beta = 0$ (panel a), $\beta = 0.5$ (panel b) and $\beta = 1$ (panel c).



Panel (a)

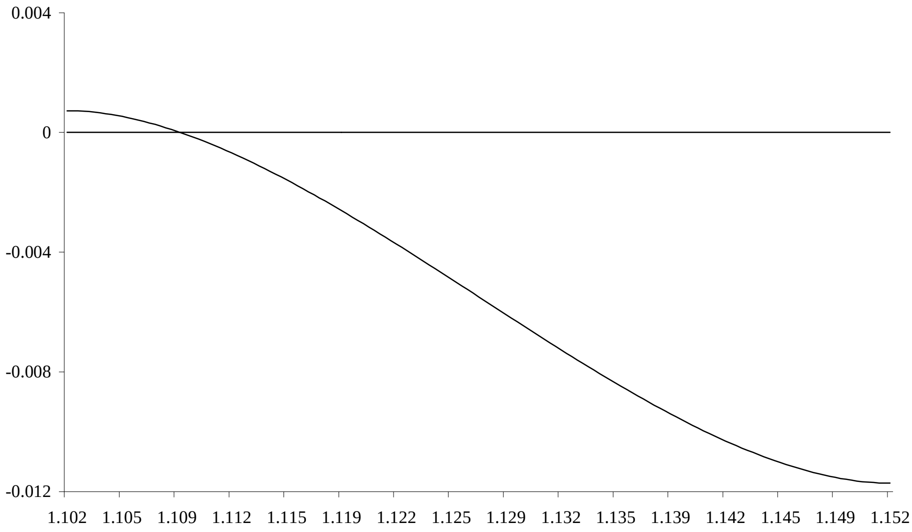


Panel (b)

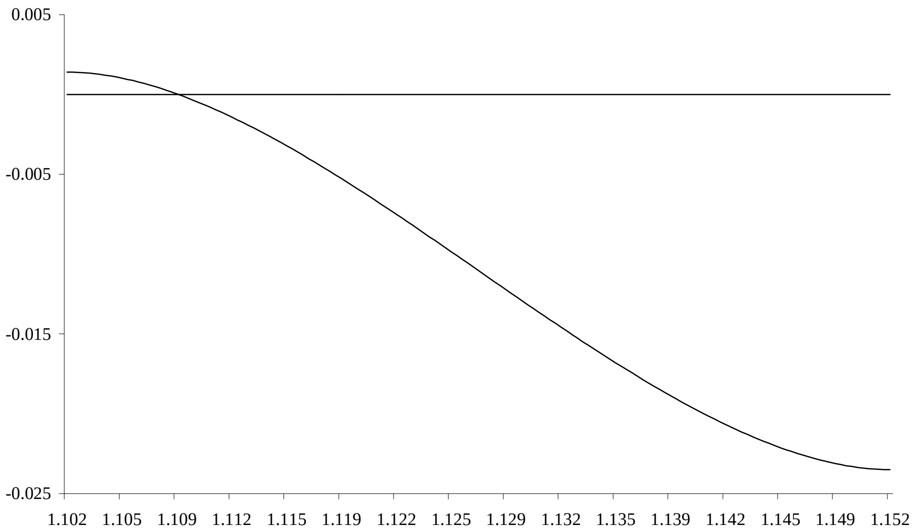


Panel (c)

Figure 3: Differenes in currency option prices: prices for $\beta = 0$ - prices for $\beta = 0.5$ (panel a) and prices for $\beta = 0$ - prices for $\beta = 1$ (panel b).



Panel (a)



Panel (b)

Figure 4: Price differences between the DJN-model and the RGBM-model for an initial exchange rate located at the centre of the lower half of the target zone.

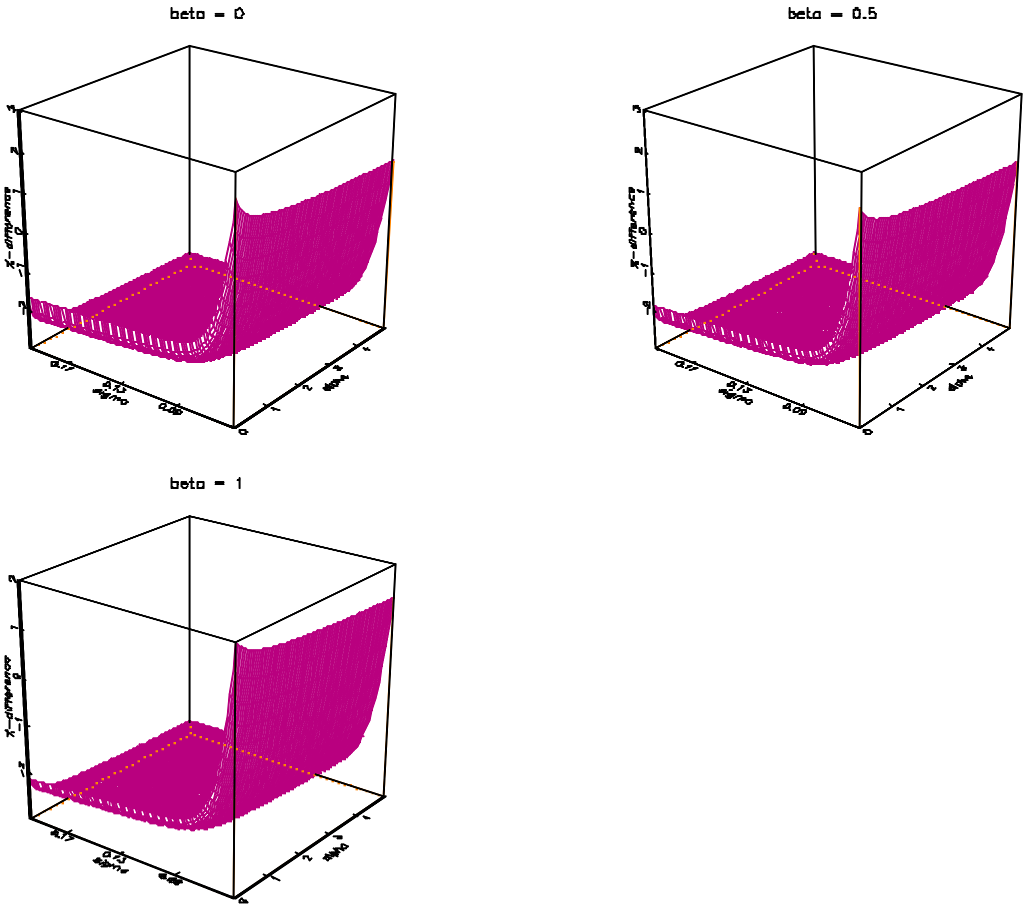


Figure 5: Price differences between the DJN-model and the RGBM-model for an initial exchange rate located at the centre of the target zone.

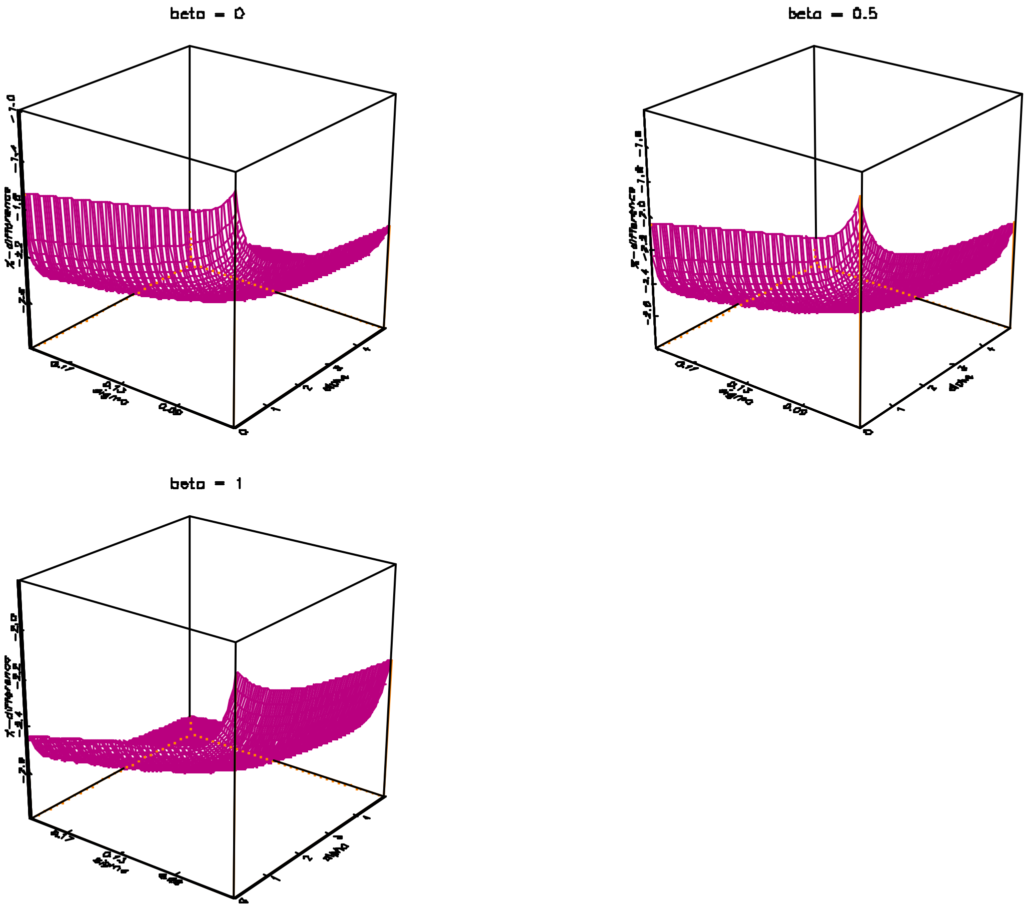


Figure 6: Price differences between the DJN-model and the RGBM-model for an initial exchange rate located at the centre of the upper half of the target zone.